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Propagation of nucleons in laser fields

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Abstract. The propagator of a spin $\frac{1}{2}$ Dirac particle with an anomalous magnetic moment in an external plane wave field is calculated by means of an off-shell eigenfunction expansion. The intensity-dependent frequency shift in Compton scattering is derived, and the modifications due to the anomalous magnetic moment are found to be non-negligible. Except in the case of a linearly polarized laser beam, there is moreover an intensity-dependent zeroth order harmonic of very low frequency, which is specific for the anomalous magnetic moment.

1. Introduction

This paper continues an earlier one where the Dirac equation in a laser field, represented by an external plane wave field, had been solved for a particle with an anomalous magnetic moment (Becker and Mitter 1974a to be referred to as I). The particle stands for a nucleon as far as the latter can be characterized by its static properties alone. The solutions obtained there differ from the Volkov solutions only by the appearance of an additional matrix factor which specifies the interaction of the magnetic moment with the laser field. Knowledge of the wavefunctions is sufficient to calculate processes where nucleons participate only as real particles, eg Compton scattering and pair creation. In all other cases, however, the propagator is needed as a starting point. These include Compton scattering with two non-laser photons and all processes involving radiative corrections.

To compute the propagator we use an off-shell eigenfunction expansion which has been applied so far only in connection with the Volkov propagator. The method is easily generalized to the present case (§ 2). It may be advantageous even in other external field problems. In § 3 we investigate the structure of the propagator as far as it is specified by general requirements. Explicit calculation follows in §§ 4 and 5. In § 6 we discuss the propagator in momentum space, the poles of which give the modified propagation law for a nucleon in the laser field. If this is known the intensity-dependent frequency shift which occurs in Compton scattering is readily obtained (§ 6).

2. An off-shell eigenfunction expansion for external field propagators

The method to be described in this section is essentially due to Ritus (1972) (see also Beers and Nickle 1972). It will be generalized here to allow for an anomalous magnetic moment term in the Dirac equation. Let us suppose the solutions $\psi(x)$ of the Dirac equation

$$(D(x)-\kappa)\psi(x) = 0 \tag{2.1}$$

$$D(x) = \mathrm{i}\not{\partial}_x - \epsilon \not{A}(x) + \frac{g'}{2}\sigma_{\mu\nu}F^{\mu\nu}(x) \tag{2.2}$$

$$g' = -\frac{e\mu}{2mc^2}, \qquad \kappa = \frac{mc}{\hbar}, \qquad \epsilon = \frac{e}{\hbar c} \tag{2.3}$$

(we use the notation of I), can be numbered in the same way as the free solutions and written in the form of a matrix applied to a free spinor

$$\psi_p(x) = E_p(x)\psi_p, \qquad (\not{p}-\kappa)\psi_p = 0. \tag{2.4}$$

The matrix $E_p(x)$ is assumed to depend on p only via scalar products with other vectors†. In order that (2.4) solves the Dirac equation (2.1) we must have

$$(D(x)-\kappa)E_p(x) = H_p(x)(\not{p}-\kappa) \tag{2.5}$$

with another matrix $H_p(x)$ which is so far unspecified but subject to the same restriction as $E_p(x)$. Multiplying (2.5) from the left with the adjoint solution $\bar{\psi}_q\bar{E}_q(x)$ where

$$\bar{E}_q(x) = \gamma^0 E_q^\dagger(x)\gamma^0$$

and integrating we find that orthogonality is expressed by

$$\int \frac{\mathrm{d}^4x}{(2\pi)^4}\bar{E}_q(x)H_p(x) = \delta(p-q). \tag{2.6}$$

Due to the above mentioned restrictions on the dependence of the matrices on p, (2.6) and (2.5) hold for arbitrary p and q even off-shell. Completeness then yields

$$\int \frac{\mathrm{d}^4p}{(2\pi)^4}E_p(x)\bar{H}_p(x') = \delta(x-x'). \tag{2.7}$$

$E_p(x)$ is assumed to include the exponential $\exp(-\mathrm{i}px)$ so that

$$D(x)E_p(x) = \not{p}E_p(x) + R.$$

We now split $E_p(x)$ into two parts

$$E_p(x) = E_p^{(1)}(x) + E_p^{(2)}(x)$$

depending on an even or odd number of γ matrices, respectively. Moving $\not{p}$ to the right we find

$$\begin{aligned}(D(x)-\kappa)E_p(x) &= (E_p^{(1)}(x) - E_p^{(2)}(x))(\not{p}-\kappa)\\ &\quad - 2\kappa E_p^{(2)}(x) + R + [\not{p}, E_p^{(1)}(x)] + \{\not{p}, E_p^{(2)}(x)\}.\end{aligned}$$

As the last four terms do not depend on $\not{p}$, (2.5) is satisfied if and only if their sum vanishes and

$$H_p(x) = \gamma_5 E_p(x)\gamma_5. \tag{2.8}$$

† To be more precise, this means the following: we choose a basis of Dirac matrices which does not explicitly contain γ_5, eg products of $\not{p}, \not{k}, \not{e}_i$ (cf (3.5)). If $\not{p}$ occurs it is moved to the right and replaced by κ.

So within (2.6) and (2.7) E_p and H_p may be interchanged and the Green function of (2.1) can be written as

$$G(x, x') = \int \frac{d^4p}{(2\pi)^4} E_p(x) \frac{\not p + \kappa}{p^2 - \kappa^2 + i\epsilon} \bar{E}_p(x') \tag{2.9}$$

which already contains the appropriate boundary condition (in this case that of time ordering).

The usefulness of (2.9) depends, of course, upon whether or not the integral can be evaluated. This is simple (apart from the algebra which is tedious) in the Volkov case of an external plane wave field which shall be considered below. The method is not restricted to Dirac equations which have solutions of nearly plane wave appearance as in the Volkov case. It applies equally well, eg for a constant magnetic field where one of the components of p has discrete values. In that case the corresponding integration has to be replaced by a summation.

To conclude we remark that exponentiating the denominator in (2.9) gives the propagator in a form which corresponds to that obtained by the proper time method (Schwinger 1951).

3. General properties of wavefunctions and propagators

From now on we specialize to the case of an external plane wave field

$$A^\mu(x) = a\, e_i^\mu A_i(\xi) \qquad \xi = kx \tag{3.1}$$

so that

$$D(x) = i\not\partial_x - \epsilon a \not e_i A_i(\xi) + ig \not k \not e_i F_i(\xi) \tag{3.2}$$

$$F_i(\xi) = \frac{dA_i(\xi)}{d\xi} \qquad g = -\frac{ea\mu}{2mc^2}. \tag{3.3}$$

The wavefunctions of this problem have been computed in (I, equation (21)). For the arbitrary constant spinor ψ_0 introduced there, we may take the free field spinor ψ_p in order to rewrite the solution in the form (2.4). The result is

$$\begin{aligned} E_p(x) &= \left(1 - \frac{\epsilon \not A \not k}{2pk}\right)\left(\hat{B}_p(\xi) - \frac{\not k \hat{\not e}_i}{pk}\kappa b_i\right) \exp\left(-ipx + \frac{i\epsilon}{2pk}\int_{\xi_0}^{\xi} d\xi'(-2pA(\xi') + \epsilon A^2(\xi'))\right) \\ &= \left(\hat{B}_{p-eA}(\xi) - \frac{\not k \hat{\not e}_i}{pk}\kappa b_i\right)\left(1 - \frac{\epsilon \not A \not k}{2pk}\right) \exp\left(\ldots\right) \end{aligned} \tag{3.4}$$

where

$$\hat{B}_p(\xi) = c_1(\xi) - c_2(\xi)\hat{\not e}_1 \hat{\not e}_2 - b_i(\xi)\hat{\not e}_i$$

$$\hat{e}_i^\mu = \hat{e}_i^\mu(p) = e_i^\mu + \frac{p_i}{pk}k^\mu \tag{3.5}$$

$$\hat{e}_i \hat{e}_j = -\delta_{ij} \qquad \hat{e}_i p = \hat{e}_i k = 0.$$

The functions $b_i(\xi)$ and $c_i(\xi)$ depend on $F_i(\xi)$ and are determined by a system of differential equations (I, equation (22); since we consider here ξ instead of u as argument, f_i has to be replaced by F_i in these equations). Explicit solutions are available for the special cases of linear polarization and of a monochromatic plane wave with circular polarization (I (36) and I (39)). The orthogonality and completeness relations (2.6) and (2.7) are fulfilled if

$$K = b_i^2 + c_i^2 = 1 \tag{3.6}$$

in contrast to I, where we had considered a three-dimensional normalization yielding $K = (2\pi)^{-3}$.

Before calculating the propagator we add some general remarks. Since the constant spinor ψ_0 in the original solution is completely arbitrary we can replace it by $\not{e}_i\psi_0$ ($i = 1, 2$). This is equivalent to the following interchanges of the functions b_i and c_i:

$$\begin{array}{lllll} \psi_0 \to \not{e}_1\psi_0: & & & & \\ & b_1 \to -c_1 & b_2 \to c_2 & c_1 \to b_1 & c_2 \to -b_2 \\ \psi_0 \to \not{e}_2\psi_0: & & & & \\ & b_1 \to -c_2 & b_2 \to -c_1 & c_1 \to b_2 & c_2 \to b_1 . \end{array} \tag{3.7}$$

The propagator, of course, must not be influenced by the interchanges (3.7), so it is allowed to depend on b_i and c_i only via the invariant bilinear combinations

$$\begin{aligned} S_1 &= b_i b_i' + c_i c_i' \\ S_{2i} &= c_1 b_i' - c_1' b_i + \epsilon_{ij}(c_2 b_j' - c_2' b_j) \\ S_3 &= c_2 c_1' - c_1 c_2' + \epsilon_{ij} b_i b_j' \end{aligned} \tag{3.8}$$

where $b_i = b_i(\xi)$, $b_i' = b_i(\xi')$ etc. As a consequence of (3.6) the functions S are normalized according to

$$S_1^2 + S_{2i}^2 + S_3^2 = 1. \tag{3.9}$$

In the case of linear polarization in the x_1 direction we obtain from I (36)

$$\begin{gathered} S_1 = K\cos(\eta-\eta') \qquad S_{21} = -K\sin(\eta-\eta') \qquad S_{22} = S_3 = 0 \\ \eta = g\int_{\xi_0}^{\xi} F(\xi')\,\mathrm{d}\xi' = g(A_1(\xi) - A_1(\xi_0)) \end{gathered} \tag{3.10}$$

and in the case of a monochromatic plane wave with circular polarization we have from I (39)

$$\begin{aligned} S_1 &= \frac{g^2 K}{\rho_1(1+4g^2)^{1/2}}\left(\cos\rho_1(\xi-\xi') + \frac{\rho_1^2}{g^2}\cos\rho_2(\xi-\xi')\right) \\ S_{21} &= \frac{2gK}{(1+4g^2)^{1/2}}\sin\left(\frac{\xi+\xi'}{2}\right)\sin\left((\rho_1-\rho_2)\frac{\xi-\xi'}{2}\right) \\ S_{22} &= \frac{2gK}{(1+4g^2)^{1/2}}\cos\left(\frac{\xi+\xi'}{2}\right)\sin\left((\rho_1-\rho_2)\frac{\xi-\xi'}{2}\right) \\ S_3 &= \frac{g^2 K}{\rho_1(1+4g^2)^{1/2}}\left(\sin\rho_1(\xi-\xi') + \frac{\rho_1^2}{g^2}\sin\rho_2(\xi-\xi')\right) \\ \rho_{1/2} &= \tfrac{1}{2}[1 \pm (1+4g^2)^{1/2}]. \end{aligned} \tag{3.11}$$

The functions S_1, S_{2i}, S_3 are, as it must be, independent of the constants of integration present in I (36) and I (39) apart from the common factor K which is determined by (3.6).

The structure of the propagator is further restricted by charge conjugation invariance which requires

$$G(x, x'|A) = CG(x', x|-A)^{\mathrm{T}}C^{-1} \tag{3.12}$$

with C the usual charge conjugation matrix, which satisfies $C\gamma_\mu^{\mathrm{T}}C^{-1} = -\gamma_\mu$. From I (26) and I (27) we infer that c_i can be chosen as even and b_i as odd upon changing the sign of g. From that we conclude that the functions S_1 and S_{2i} are symmetric and S_3 is antisymmetric under the combined interchange $x \leftrightarrow x'$ and $g \to -g$. A further symmetry which reflects the reality of $A_\mu(\xi)$ reads

$$G(x, x'|A, \kappa) = -\gamma_0 BG(x, x'|-A, -\kappa)^*(\gamma_0 B)^{-1} \tag{3.13}$$

where B is the Dirac matrix connected with time reversal and satisfying $B\gamma_\mu^* B^{-1} = \gamma_0\gamma_\mu\gamma_0$. $\kappa \to -\kappa$ is understood to include $g \to -g$ via (3.3) so that all the functions S are symmetric with respect to $A \to -A$ and $\kappa \to -\kappa$.

At least in the case of the neutron, where the only non-trivial dependence on ξ and ξ' is given by the functions S, the propagator is determined by (3.12) and (3.13) to a large extent.

4. The neutron propagator

In order to split the calculation of the complete propagator into manageable pieces we compute at first the propagator of a neutral particle with an anomalous magnetic moment. Exponentiating the denominator in (2.9) we have

$$\begin{aligned} G_N(x, x') = {} & \mathrm{i}\int_0^\infty \mathrm{d}s \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \exp[\mathrm{i}s(p^2-\kappa^2+\mathrm{i}\epsilon)] \\ & \times\left(\hat{B}_p(\xi) - \frac{\not{k}\not{\hat{e}}_i}{pk}\kappa b_i(\xi)\right)(\kappa + \mathrm{i}\not{\partial}_z) \\ & \times\left(\bar{\hat{B}}_p(\xi') + \frac{\not{k}\not{\hat{e}}_j}{pk}\kappa b_j(\xi')\right)\mathrm{e}^{-\mathrm{i}pz} \qquad z = x - x'. \end{aligned} \tag{4.1}$$

The differentiation with respect to z refers only to the explicit dependence on this variable. In the absence of the non-trivial part of the exponential in (3.4) the integration with respect to p is straightforward if performed in terms of the light-like components p_u, p_v and p_i (cf I (8)). The result is

$$\begin{aligned} G_N(x, x') = {} & -\frac{1}{16\pi^2}\int \frac{\mathrm{d}s}{s^2}\mathrm{e}^{-\mathrm{i}s\kappa^2}\left(\hat{B}_\partial(\xi) - \frac{2s\kappa}{kz}\not{k}\not{\hat{e}}_i b_i(\xi)\right)\left(\kappa + \frac{\not{z}}{2s}\right) \\ & \times\left(\bar{\hat{B}}_\partial(\xi') + \frac{2s\kappa}{kz}\not{k}\not{\hat{e}}_j b_j(\xi')\right)\exp\left(-\mathrm{i}\frac{z^2}{4s}\right). \end{aligned} \tag{4.2}$$

$\hat{e}_i$ is now given by

$$\hat{e}_i^\mu = e_i^\mu - \mathrm{i}\frac{2s}{kz}\frac{\partial}{\partial z_i}k^\mu$$

which has been indicated by writing $\hat{B}_{\partial}$. The differentiations refer again to the explicit dependence on z throughout the whole integrand. The propagator becomes

$$G_N(x,x') = -\frac{1}{16\pi^2}\int_0^\infty \frac{\mathrm{d}s}{s^2}\exp\left[-\mathrm{i}\left(s\kappa^2+\frac{z^2}{4s}\right)\right]$$
$$\times\left[\left(\kappa+\frac{\not{z}}{2s}\right)(S_1-\not{\hat{e}}_1\not{\hat{e}}_2S_3)-\frac{\not{z}\not{\hat{e}}_i}{2s}S_{2i}-\frac{2_i}{kz}\gamma_5\not{k}S_3\right.$$
$$\left.+\frac{\not{k}\not{\hat{e}}_i}{kz}(2s\kappa^2-\mathrm{i})S_{2i}-\mathrm{i}\kappa kz\epsilon_{ij}\gamma_5\not{\hat{e}}_jS_{2i}\right] \tag{4.3}$$

where now

$$\hat{e}_i^\mu = e_i^\mu+\frac{z_i}{kz}k^\mu. \tag{4.4}$$

An explicit form may be obtained by means of the integral representations

$$-\frac{1}{16\pi^2}\int_0^\infty \frac{\mathrm{d}s}{s^u}\exp\left[-\mathrm{i}\left(s\kappa^2+\frac{z^2}{4s}\right)\right]$$
$$=\frac{\mathrm{i}^{2-u}\kappa^{2u-2}}{2^{5-u}\pi}\,\frac{H_{u-1}^{(2)}((\kappa^2z^2-\mathrm{i}\epsilon)^{1/2})}{(\kappa^2z^2-\mathrm{i}\epsilon)^{(u-1)/2}} \qquad (u=1,2)$$

$$-\frac{1}{16\pi^2}\int_0^\infty \frac{\mathrm{d}s}{s^3}\exp\left[-\mathrm{i}\left(s\kappa^2+\frac{z^2}{4s}\right)\right]$$
$$=\frac{\mathrm{i}\kappa^4}{4\pi(\kappa^2z^2-\mathrm{i}\epsilon)^{1/2}}\left(H_0^{(2)}((\kappa^2z^2-\mathrm{i}\epsilon)^{1/2})-\frac{H_1^{(2)}((\kappa^2z^2-\mathrm{i}\epsilon)^{1/2})}{(\kappa^2z^2-\mathrm{i}\epsilon)^{1/2}}\right). \tag{4.5}$$

For subsequent calculations it is, however, often more convenient to start from the integral form (4.3).

5. The complete propagator

By means of (2.9) the complete propagator is calculated by transforming the free one with the complete wavefunctions. In a quite similar way it can be computed by transforming the neutron propagator with the pure Volkov wavefunctions. The first version of (3.4) can be rewritten in the following form:

$$E_p(x) = V_p(x)N_p(x) \tag{5.1}$$

$$V_p(x) = \left(1-\frac{\epsilon\not{A}\not{k}}{2pk}\right)\exp\left(\frac{\mathrm{i}\epsilon}{2pk}\int_{\xi_0}^{\xi}\mathrm{d}\xi'\,(-2pA(\xi')+\epsilon A^2(\xi'))\right) = \gamma_5V_p(x)\gamma_5 \tag{5.2}$$

$$N_p(x) = \left(\hat{B}_p(\xi)-\frac{\not{k}\not{\hat{e}}_i}{pk}\kappa b_i(\xi)\right)\mathrm{e}^{-\mathrm{i}px} \tag{5.3}$$

where $V_p(x)\,\mathrm{e}^{-\mathrm{i}px}\psi_p$ is the Volkov wavefunction and $N_p(x)\psi_p$ the neutron wavefunction. The neutron propagator of (4.3) can be unambiguously written as

$$G_N(x,x') = G_N(\xi,\xi',z) \qquad z = x-x'$$

where z stands for the explicitly appearing difference $x-x'$ as exhibited in (4.3). If we now insert (5.1) into (2.9) we obtain

$$G(x,x') = \int \frac{d^4p\, d^4q\, d^4a}{(2\pi)^8} e^{ia(p-q)}\, V_p(x) N_q(x) \frac{\not{q}+\kappa}{q^2-\kappa^2+i\epsilon} \bar{N}_q(x')\bar{V}_p(x')$$

$$= \int \frac{d^4p\, d^4a}{(2\pi)^4} e^{iap}\, V_p(x) G_N(\xi, \xi', z+a) \bar{V}_p(x')$$

$$= \int \frac{d^4p\, d^4a}{(2\pi)^4} e^{i(a-z)p}\, V_p(x)\, G_N(\xi, \xi', a) \bar{V}_p(x'). \tag{5.4}$$

The evaluation of (5.4) is straightforward, although rather tedious, and no means are required in addition to those used for the calculation of the neutron propagator. We only give the final result

$$G(x,x') = -\frac{1}{16\pi^2}\phi(x,x') \int_0^\infty \frac{ds}{s^2} \exp\left[-is\left(m^2+\frac{z^2}{4s}\right)\right]$$
$$\times \left\{\left(\kappa+\frac{\not{z}}{2s}\right)(S_1 - \hat{e}_1\hat{e}_2 S_3) + kz(M_i + i\gamma_5 M_k \epsilon_{ki})S_{2i} + \tfrac{1}{2}[\not{z},\not{k}]L_i S_{2i}\right.$$
$$-\frac{1}{2s}\not{z}\hat{e}_i S_{2i} + kz\hat{e}_1\hat{e}_2 L_k \epsilon_{ki} S_{2i} + ikz\gamma_5 \hat{e}_i\left(L_i S_3 - \epsilon_{ik} M_k S_1 + \frac{\kappa}{kz}\epsilon_{ij} S_{2j}\right)$$
$$-kz\hat{e}_i(L_i S_1 + \epsilon_{ik} M_k S_3) + s\not{k}(RS_1 + i\hat{R}S_3 + 2\kappa M_i S_{2i})$$
$$+ is\gamma_5 \not{k}\left[i\hat{R}S_1 + 2\left(\frac{i}{skz} - kzL_i^2 + \frac{N}{2}\right)S_3 - 2\kappa L_k \epsilon_{ki} S_{2i}\right]$$
$$+ s\not{k}\hat{e}_i\left[2\kappa L_k \epsilon_{ki} S_3 - 2\kappa M_i S_1 + \left(\frac{2\kappa^2}{kz} - \frac{i}{skz} - R\right)S_{2i}\right.$$
$$\left.\left. + 2kz(L_i L_j - M_i M_j)S_{2j}\right]\right\}. \tag{5.5}$$

The ingredients of (5.5) are those of the Volkov propagator (Brown and Kibble 1964) which is recovered by setting $S_1 = 1, S_{2i} = S_3 = 0$. In particular ϕ denotes the usual gauge-dependent line integral and m^2 the positive field dependent mass squared.

$$\phi(x,x') = \exp\left(i\epsilon a \frac{(x_i - x_i')}{\xi-\xi'} \int_{\xi'}^{\xi} A_i(\eta)\, d\eta\right)$$

$$m^2 = \kappa^2 + \frac{\epsilon^2 a^2}{\xi-\xi'} \int_{\xi'}^{\xi} d\eta\, A_i(\eta)\left(A_i(\eta) - \frac{1}{\xi-\xi'}\int_{\xi'}^{\xi} d\eta'\, A_i(\eta')\right)$$

$$M_i = -\frac{\epsilon a}{2(\xi-\xi')}(A_i(\xi) - A_i(\xi'))$$

$$L_i = \frac{\epsilon a}{2(\xi-\xi')}\left(A_i(\xi) + A_i(\xi') - \frac{2}{\xi-\xi'}\int_{\xi'}^{\xi} d\eta\, A_i(\eta)\right) \tag{5.6}$$

$$N = \frac{1}{2}\left(\frac{\partial}{\partial\xi} - \frac{\partial}{\partial\xi'}\right)m^2$$

$$R = N - 2(\xi-\xi')M_i^2$$

$$\hat{R} = 2i(\xi-\xi')M_i \epsilon_{ij} L_j.$$

(The notation used here is that of Becker and Mitter (1974b), where one finds an extensive discussion of these functions, especially in the case of circular polarization.) The integral over s in (5.5) can be performed as above according to (4.5).

6. Propagation of nucleons and intensity-dependent frequency shift in Compton scattering

The propagator which we have arrived at in the last section has a structure very similar to the Volkov propagator. In fact, the various terms of the latter or similar ones have merely been multiplied by one of the functions S. The impact of this modification is most easily seen from the Fourier transform

$$\tilde{G}(p,p') = \int \frac{\mathrm{d}^4x\,\mathrm{d}^4x'}{(2\pi)^4} \exp(\mathrm{i}px - \mathrm{i}p'x')G(x,x'). \tag{6.1}$$

In the following we treat the case of circular polarization which is particularly simple because

$$m^2 = \kappa^2 + \epsilon^2 a^2 \left(1 - \frac{\sin^2 \frac{1}{2}(\xi - \xi')}{[\frac{1}{2}(\xi - \xi')]^2}\right) \tag{6.2}$$

depends only on the coordinate difference. By the way we remark that the propagator (5.5) simplifies a bit in that case, since $L_i S_{2i} = M_i \epsilon_{ij} S_{2j} = 0$ and $M_i S_{2i}$ as well as $L_i \epsilon_{ij} S_{2j}$ depend only on the coordinate difference.

We shall write down the Fourier transforms of two typical terms, namely

$$\{G_1(x,x'), G_2(x,x')\} = -\frac{1}{16\pi^2}\phi(x,x') \int \frac{\mathrm{d}s}{s^2} \exp\left[-\mathrm{i}s\left(m^2 + \frac{z^2}{4s^2}\right)\right]\{S_1, S_{21}\}$$

which are

$$\begin{aligned}\tilde{G}_1(p,p') = {} & \frac{g^2}{2\rho_1(1+4g^2)^{1/2}} \sum_{l=-\infty}^{\infty} \delta(p - p' + lk)\,\mathrm{e}^{\mathrm{i}rl} \\ & \times \sum_{n=-\infty}^{\infty} \left(\frac{J_{n-1}(\zeta)J_{l+n-1}(\zeta) + (\rho_1^2/g^2)J_n(\zeta)J_{l+n}(\zeta)}{[p-(n-\rho_2)k]^2 - \kappa_*^2}\right. \\ & \left. + \frac{J_{n+1}(\zeta)J_{l+n+1}(\zeta) + (\rho_1^2/g^2)J_n(\zeta)J_{l+n}(\zeta)}{[p-(n+\rho_2)k]^2 - \kappa_*^2}\right)\end{aligned} \tag{6.3}$$

and

$$\begin{aligned}\tilde{G}_2(p,p') = {} & \frac{g}{2(1+4g^2)^{1/2}} \sum_{l=-\infty}^{\infty} \delta(p - p' + lk)\,\mathrm{e}^{\mathrm{i}rl} \\ & \times \sum_{n=-\infty}^{\infty} \left(\frac{\mathrm{e}^{\mathrm{i}r}J_{n+l+1}(\zeta)J_n(\zeta) - \mathrm{e}^{-\mathrm{i}r}J_{n+l}(\zeta)J_{n+1}(\zeta)}{[p-(n+\rho_2)k]^2 - \kappa_*^2}\right. \\ & \left. + \frac{\mathrm{e}^{-\mathrm{i}r}J_{n+l-1}(\zeta)J_n(\zeta) - \mathrm{e}^{\mathrm{i}r}J_{n+l}(\zeta)J_{n-1}(\zeta)}{[p-(n-\rho_2)k]^2 - \kappa_*^2}\right)\end{aligned} \tag{6.4}$$

where

$$\kappa_*^2 = \kappa^2 + \epsilon^2 a^2 \tag{6.5}$$

is the well known effective mass and r and ζ are given by

$$\zeta\, e^{ir} = -\frac{\epsilon a}{pk} p(e_1 + ie_2). \tag{6.6}$$

If $g = 0$, (6.3) reduces to the Volkov result

$$\tilde{G}_1(p, p')|_{g=0} = \sum_{l=-\infty}^{\infty} \delta(p-p'+lk)\, e^{irl} \sum_{n=-\infty}^{\infty} \frac{J_{l+n}(\zeta)J_n(\zeta)}{(p-nk)^2-\kappa_*^2} \tag{6.7}$$

(cf Reisz and Eberly 1966, Becker and Mitter 1974b). Thus the main effect of the anomalous magnetic moment is to replace the Volkov quasi-energy levels

$$(p-nk)^2 = \kappa_*^2 \tag{6.8}$$

by

$$[p-(n\pm\rho_2)k]^2 = \kappa_*^2 \tag{6.9}$$

These are the only poles which occur in the momentum transform of (5.5). One can easily convince oneself that all of the terms of (5.5) have transforms similar to (6.3) and (6.4), even if they include the M_i, N etc which are simple trigonometric functions. Because

$$\rho_2 = \tfrac{1}{2}[1-(1+4g^2)^{1/2}] \simeq -g^2$$

the modification to (6.8) is very small.

In the case of the neutron the parameter ζ vanishes. Therefore only one term contributes to the double sums in (6.3) and (6.4). $\tilde{G}_1(p, p')$ becomes diagonal, $l = 0$ and $n = \pm 1$ or 0. This had to be expected because the line integral $\phi(x, x')$ accounts for all non-diagonal terms except $l = \pm 1$, and $\phi(x, x') = 1$ for the neutron. So $G_1(x, x')$ depends only on the coordinate difference. $\tilde{G}_2(p, p')$ has only off-diagonal terms with $l = \pm 1$, $n = 0, \mp 1$. All terms of the neutron propagator (4.3) have similar properties.

Once the quasi-energy levels (6.9) are known it is not difficult to calculate the intensity-dependent frequency shift, which occurs in Compton scattering off a particle with an anomalous magnetic moment. In this connection Compton scattering is understood to mean the net absorption of one or more quanta from the laser beam together with the emission of one photon with arbitrary momentum k'. The process could equally well be referred to as bremsstrahlung in the external field.

We proceed in a similar way as Brown and Kibble (1964). Momentum conservation reads

$$p + rk = p' + k' \tag{6.10}$$

where p and p' are the initial and final momenta, respectively, of the proton (or neutron), k' is the momentum of the emitted photon and r is the net number of absorbed laser quanta. Of course, if $p^2 = p'^2 = \kappa^2$, the usual Compton formula results. To obtain an intensity-dependent frequency shift one has to take into account the modified propagation law (6.9). We then have to relate the proton momenta inside the laser beam to those outside:

$$p = q + \left(l \pm \rho_2 + \frac{\kappa_*^2 - \kappa^2}{2qk}\right)k$$

$$p' = q' + \left(l' \pm \rho_2 + \frac{\kappa_*^2 - \kappa^2}{2q'k}\right)k \tag{6.11}$$

where now $q^2 = q'^2 = \kappa^2$ and the laboratory frame is defined by $q_0 = \kappa$, $\boldsymbol{q} = 0$. The sign of ρ_2 may be different in the expressions for p and p'. Obviously, the *ansatz* (6.11) satisfies (6.9). It is reasonable to assume that the difference between inside and outside momenta is proportional to k, since this is the only direction where translational invariance is violated. From (6.10) and (6.11) we obtain the frequency of the emitted photon in the laboratory frame

$$\omega' = (\bar{r}+2\bar{\rho})\omega\left[1+\left(\frac{\kappa_*^2-\kappa^2}{\kappa^2}+2(\bar{r}+2\bar{\rho})\frac{\omega}{\kappa}\right)\sin^2\left(\frac{\vartheta}{2}\right)\right]^{-1} \tag{6.12}$$

where ϑ is the scattering angle of the emitted photon, so that $kk' = 2\sin^2(\vartheta/2)$, and

$$\bar{r} = r+l-l' \qquad \bar{\rho} = \pm\rho_2 \text{ or } 0.$$

(6.12) is valid for protons as well as neutrons. In the latter case, however, $\bar{r}$ is restricted to the values $\bar{r} = 0, 1, 2$.

7. Conclusions

The calculation of the frequency shift could have been done equally well by means of the wavefunctions without making any use of the propagator. We have seen, however, that it is immediately obtained from the poles of the propagator in momentum space.

We turn now to discuss the frequency shift (6.12). It differs from the result of Brown and Kibble by the replacement of $\bar{r}$ by $\bar{r}+2\bar{\rho}$. Hence, one effect of the anomalous magnetic moment is to split the energy levels of the scattered photon into triplets. The relative level spacing is approximately $\Delta\omega/\omega = 2\rho_2$. The usual parameter characterizing the intensity of the laser is $\nu = \epsilon a/(m_e c^2)$ with m_e the mass of the electron. At present this value is at best $\nu \lesssim 1$, so we have $|\rho_2| \lesssim 10^{-6}$ due to the electron nucleon mass ratio. This is of the same order of magnitude as the second term in the denominator of (6.12), so the modification of the intensity-dependent frequency shift by the anomalous magnetic moment is quite considerable. For a high intensity ($\nu \lesssim 1$) laser with say $\lambda \sim 10^{-6}$m we have $\omega/\kappa \sim 10^{-9}$. Hence the third term in the denominator of (6.12) which gives the usual Compton scattering frequency shift can be completely neglected.

The frequency shift (6.12) exhibits one distinguishing feature: ω' does not vanish if $\bar{r} = 0$. So in addition to the $\bar{r}$th order harmonics ($\bar{r} = 1, 2, \ldots$) discussed so far, there is a very low frequency (in the radio range under the same conditions as above) which is entirely due to the presence of the anomalous magnetic moment. Very crude estimates based on phase space considerations analogous to Brown and Kibble indicate that the cross section for scattering into this level is of the same order of magnitude as first harmonic scattering. There might be, however, various cancellations with the tendency to reduce the cross section, and more detailed calculations should be carried out before quantitative statements can be reached.

So far we have investigated only circular polarization and found all effects to be at best of order g^2. Since in the case of linear polarization the parameter g enters (3.10) linearly, one might expect a larger effect on the frequency shift. There is, however, no effect at all. The reason is that the poles of the propagator are now given by (6.8) (with $\kappa_*^2 = \kappa^2 + \frac{1}{2}\epsilon^2 a^2$) as in the case without anomalous magnetic moment. This is not hard to understand: the level splitting is due to different orientations of the magnetic moment. If the magnetic field alternates in a definite direction all orientations are equivalent.

In so far as the electron's anomalous magnetic moment can be taken into account by merely adding the σF term in the Dirac equation the formulae given above apply to the electron as well. The parameter g^2 is of the same order of magnitude in both cases. For the electron, however, $(\kappa_*^2 - \kappa^2)/\kappa^2 = \nu^2 \lesssim 1$.

We remark finally that it may be a very difficult question to what extent the intensity-dependent frequency shift can be observed in actual experiments. More realistic but still very idealized wave packet calculations (Dawson and Fried 1970, Neville and Rohrlich 1971) indicate that no definite shifted frequency can be expected but rather a considerably broadened and complicated lineshape.

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Note added in proof. The optimistic estimate on the magnitude of the zeroth order harmonic is not sustained by explicit calculation (see a forthcoming paper by W Becker, V Koch and H Mitter).

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